## LITERATURE CITED

1. I. A. Kuznetsova, A. M. Andreev, Yu. A. Sokovishin, and V. F. Stepanov, Inzh.-Fiz. Zh., 16, No. 5 (1969).
2. R. K. Irey, Trans. ASME, 90C, No. 1 (1968).
3. I. E. Zino, Inzh.-Fiz. Zh., 28, No. 2 (1975).
4. H. S. Carslaw and J. C. Jaeger, Conduction of Heat in Solids, 2nd ed., Oxford University Press (1959).
5. N. N. Lebedev, Special Functions and Their Application [in Russian], Fizmatgiz (1963).
6. V. P. Isachenko, V. A. Osipova, and A. S. Sukomel, Heat Transfer [in Russian], Énergiya, Moscow (1969).
7. I. E. Zino and Yu. A. Sokovishin, Inzh.-Fiz. Zh., 26, No. 2 (1974).


# TEMPERATURE CONDITIONS OF ROCK EXCAVATION

B. A. Krasovitskii and F. S. Popov

UDC 622.536.24:536.244

Heat exchange is considered between ventilating air and mining rocks in the case of variable air temperature at the rock excavation entrance. Formulas are given for the temperature of the ventilating air according to the extent of exploitation.

In developing deep pits with seams of high temperature as well as in many pits in the Far North that have seams of low temperature the need arises for regulating the temperature of the ventilating air. The problem thus arises of determining the temperature of the ventilating air along the length of the underground excavation at different time instants. An exact solution of this problem which can be obtained by using the operational calculus is very cumbersome. Several hours of machine time are needed to set it on an M-220 electronic computer. Numerous approximation methods have been proposed to find the solution. It was proposed in [1] that the nonstationary heat exchange be taken into account between the air and the mined rocks with the aid of a coefficient of nonstationary heat exchange; to determine the latter a dependence was assumed which was an approximation to the exact solution. In [2] the formula for the nonstationary exchange coefficient was obtained by approximating the solution of the problem under consideration on a hydrointegrator. In the present article the integral method [3] is used to solve the heat-exchange problem between the ventilating air and the mined rocks which, as shown below, produces a good agreement with the exact solution. The solution is obtained for the case of variable air temperature at the mining operation entrance.

The equation of the heat flow of the ventilating air at the production face is given by

$$\rho_a c_a v \frac{\partial T}{\partial z} = \frac{2}{r_0} \lambda \frac{\partial \theta}{\partial \xi}\Big|_{\xi=r_0} + \frac{q}{\pi r_0^2}, \tag{1}$$

$$T|_{z=0} = T_0(t). \tag{2}$$

In the above, q denotes the power of the internal heat sources per one meter of output face. Here heat emission due to moisture condensation or to various operating devices, etc., can also be included.

The heat-conduction equation for the rock mass surrounding the face is

$$\frac{\partial \theta}{\partial t} = a\left(\frac{1}{\xi}\cdot\frac{\partial \theta}{\partial \xi} + \frac{\partial^2 \theta}{\partial \xi^2}\right), \tag{3}$$

$$\theta|_{\tau=0} = \theta_M,$$

$$\lambda \frac{\partial \theta}{\partial \xi}\Big|_{\xi=r_0} = \left(\frac{1}{\alpha} + \sum_n \frac{\delta_n}{\lambda_n}\right)^{-1} (\theta|_{\xi=r_0} - T). \tag{4}$$

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 31, No. 2, pp. 339-346, August, 1976. Original article submitted August 27, 1974.

The usual assumptions were made when writing down the equations: the inertia term in the heat-flow equation is small for the air; the axial heat flow in the rock mass is small compared with the radial one; heat transfer takes place in the air only by means of convective heat exchange. The following dimensionless variables are introduced:

$$r=\frac{\xi}{r_0},\ z=\frac{\bar{z}}{L},\ t=\frac{\bar{t}a}{r_0^2}\equiv\mathrm{Fo}.$$

Substituting the above in (1)-(4), the following system of equations is obtained:

$$B\frac{\partial T}{\partial z}=\left.\frac{\partial\theta}{\partial r}\right|_{r=1}+Q, \tag{5}$$

$$T|_{z=0}=T_0(t), \tag{6}$$

$$\frac{\partial\theta}{\partial t}=\frac{1}{r}\cdot\frac{\partial\theta}{\partial r}+\frac{\partial^2\theta}{\partial r^2}, \tag{7}$$

$$\theta|_{t=0}=\theta_{\mathrm{M}}, \tag{8}$$

$$\left.\frac{\partial\theta}{\partial r}\right|_{r=1}=\alpha(\theta|_{r=1}-T). \tag{9}$$

In the above

$$B=\frac{c_a\rho_a r_0^2 v}{2L\lambda};\ Q=\frac{q}{2\pi\lambda};\ \alpha=\frac{r_0}{\lambda\left(\frac{1}{\bar{\alpha}}+\sum_n\frac{\delta_n}{\lambda_n}\right)}.$$

It is assumed from now on that the function $T_0(t)$ is given in the following form:

$$T_0(t)=\begin{cases}T_{10} & t_0=0\leqslant t<t_1,\\ T_{20} & t_1\leqslant t<t_2,\\ \cdots & \cdots\\ T_{i0} & t_{i-1}\leqslant t<t_i,\\ \cdots & \cdots\end{cases} \tag{10}$$

This is the best way to specify the function $T_0(t)$ if the temperature is predicted in the case of the air supply in the mine having its natural temperature or if it differs from the temperature of the air outside by a known quantity (in the case of heating up). In the latter case the intervals of the dimensionless time $[t_0, t_1]$, $[t_1, t_2]$, etc., may represent, for example, months, and $T_{10}, T_{20},\ldots$, the predicted monthly average temperatures of the outside air. Other variables are now introduced, namely,

$$\bar{T}=T-\frac{Q}{B}z, \tag{11}$$

$$\bar{\theta}=\theta-\frac{Q}{B}z. \tag{12}$$

For these variables the problem (5)-(9) can be written in the form

$$B\frac{\partial\bar{T}}{\partial z}=\left.\frac{\partial\bar{\theta}}{\partial r}\right|_{r=1}, \tag{13}$$

$$\frac{\partial\bar{\theta}}{\partial t}=\frac{1}{r}\cdot\frac{\partial\bar{\theta}}{\partial r}+\frac{\partial^2\bar{\theta}}{\partial r^2}, \tag{14}$$

$$\bar{T}|_{z=0}=T_0(t), \tag{15}$$

$$\left.\frac{\partial\bar{\theta}}{\partial r}\right|_{r=1}=\alpha\left(\bar{\theta}|_{r=1}-\bar{T}\right), \tag{16}$$

$$\bar{\theta}|_{t=0}=\theta_{\mathrm{M}}-\frac{Q}{B}z. \tag{17}$$

The functions $\bar{T}$ and $\bar{\theta}$ are represented as sums of the following functions:

$$\bar{T}=\bar{T}_1+\bar{T}_2,\ \bar{\theta}=\bar{\theta}_1+\bar{\theta}_2.$$

The functions $\overline{\theta}_i$ and $\overline{T}_i$ are sought as solutions of the following problems:

$$B\,\frac{\partial \overline{T}_1}{\partial z}=\left.\frac{\partial \overline{\theta}_1}{\partial r}\right|_{r=1}, \tag{18}$$

$$\frac{\partial \overline{\theta}_1}{\partial t}=\frac{1}{r}\cdot\frac{\partial \overline{\theta}_1}{\partial r}+\frac{\partial^2 \overline{\theta}_1}{\partial r^2}, \tag{19}$$

$$\overline{T}_1|_{z=0}=T_0(t), \tag{20}$$

$$\left.\frac{\partial \overline{\theta}_1}{\partial r}\right|_{r=1}=\alpha\left(\overline{\theta}_1-\overline{T}_1\right), \tag{21}$$

$$\overline{\theta}_1|_{t=0}=0, \tag{22}$$

$$B\,\frac{\partial \overline{T}_2}{\partial z}=\left.\frac{\partial \overline{\theta}_2}{\partial r}\right|_{r=1}, \tag{23}$$

$$\frac{\partial \overline{\theta}_2}{\partial t}=\frac{1}{r}\cdot\frac{\partial \overline{\theta}_2}{\partial r}+\frac{\partial^2 \overline{\theta}_2}{\partial r^2}, \tag{24}$$

$$\overline{T}_2|_{z=0}=0, \tag{25}$$

$$\left.\frac{\partial \overline{\theta}_2}{\partial r}\right|_{r=1}=\alpha\left(\overline{\theta}_2-\overline{T}_2\right), \tag{26}$$

$$\overline{\theta}_2|_{t=0}=\theta_{\text{M}}-\frac{Q}{B}\,z. \tag{27}$$

It is obvious that the sums $\overline{T}_1+\overline{T}_2$ and $\overline{\theta}_1+\overline{\theta}_2$ of the respective solutions of the problems (18)-(22) and (23)-(24) will be the solutions of the problems (13)-(17).

The problem (18)-(22) is considered first. Let the functions $P(z,\,t,\,\tau,\,y)$ and $F(r,\,z,\,t,\,\tau,\,y)$ be solutions of the problem

$$B\,\frac{\partial P}{\partial z}=\left.\frac{\partial F}{\partial r}\right|_{r=1}, \tag{28}$$

$$\frac{\partial F}{\partial t}=\frac{1}{r}\cdot\frac{\partial F}{\partial r}+\frac{\partial^2 F}{\partial r^2}, \tag{29}$$

$$F|_{t\leqslant\tau}=0, \tag{30}$$

$$\left.\frac{\partial F}{\partial r}\right|_{r=1}=\alpha\left(F|_{r=1}-P\right), \tag{31}$$

$$P|_{z=0}=\delta(t-\tau)\,y, \tag{32}$$

where

$$\delta(x)=\begin{cases}1 & \text{for } x\geqslant 0,\\ 0 & \text{for } x<0.\end{cases}$$

Let us consider the functions

$$\Pi_i=P(z,\,t,\,t_{i-1},\,T_{i0})-P(z,\,t,\,t_i,\,T_{i0}),$$
$$\Phi_i=F(r,\,z,\,t,\,t_{i-1},\,T_{i0})-F(r,\,z,\,t,\,t_i,\,T_{i0}).$$

These functions satisfy Eqs. (28) and (29), the boundary condition (31), and also the following boundary conditions:

$$\Pi_i|_{z=0}=\begin{cases}T_{i0} & \text{for } t_{i-1}\leqslant t<t_i,\\ 0 & \text{for } t\notin[t_{i-1},\,t_i],\end{cases} \tag{33}$$

$$\Pi_i|_{t<t_{i-1}}=0, \tag{34}$$

$$\Phi_i|_{t\leqslant t_{i-1}}=0. \tag{35}$$

Hence it follows that the solution of the problem (18)-(22) is of the form

$$\overline{T}_1(z,\,t)=\sum_{i=1}^{k}\Pi_i, \tag{36}$$

$$\overline{\theta}_1(r, z, t) = \sum_{i=1}^{k} \Phi_i, \tag{37}$$

where k is such that $t_k < t < t_{k+1}$. To obtain an approximate solution of the problem (29)-(31) the integral method of Goodman [3] is applied. To this end one introduces the time-dependent radius R(t) of the heat effect; by definition one sets on this radius

$$F|_{r=R(t)} = 0, \tag{38}$$

$$\left.\frac{\partial F}{\partial r}\right|_{r=R(t)} = 0. \tag{39}$$

Integrating the heat-conduction equation (29) with respect to the space coordinate from 1 to R, one obtains

$$\frac{\partial}{\partial t}\int_1^R rFdr = -\left.\frac{\partial F}{\partial r}\right|_{r=1}. \tag{40}$$

The profile F(r, t) is specified in the form

$$F(r, t) = a_1 \ln r + a_2 + a_3 r. \tag{41}$$

In the above $a_1$, $a_2$, $a_3$ are some unknown time functions. By satisfying the boundary conditions (31), (38), (39) one finds

$$a_1 = \frac{-\alpha P}{1 + \dfrac{\alpha - 1}{R} + \alpha(\ln R - 1)},$$
$$a_2 = a_1(1 - \ln R),\ a_3 = -\frac{a_1}{R}. \tag{42}$$

Substituting the profile (41) together with the coefficients (42) into the relations (40) one arrives at an ordinary differential equation of the first order for the heat effect radius, namely,

$$\dot{R} = \frac{12\left(\dfrac{1}{R} - 1\right)\left[1 + \dfrac{\alpha - 1}{R} + \alpha(\ln R - 1)\right]}{3(1-\alpha) + \dfrac{3}{R^2}\left(\alpha - \dfrac{3}{7}\right) + R(3\alpha - 2) + \dfrac{3}{R}(2-\alpha) + \dfrac{2\ln R}{R^2}(\alpha - 3) - 2\alpha R \ln R}, \tag{43}$$

$$R|_{t=\tau} = 1. \tag{44}$$

Integrating numerically (43) one finds the function R(t − τ). In the neighborhood of the point R = 1, Eq. (43) becomes

$$\dot{R} = \frac{3}{R-1} + o(R-1)^3.$$

Its solution which satisfies the initial condition (44) is as follows:

$$R = 1 + \sqrt{6(t - \tau)}. \tag{45}$$

Thus when integrating numerically one has to adopt as the initial condition in Eq. (43)

$$R|_{t=t_0} = 1 + \sqrt{6(t_0 - \tau)}, \tag{46}$$

where $t_0$ is such that $R^3(t_0 - \tau) < 1 + \varepsilon$, ε is an admissible integration error. The estimates show that for a wide range of the values of α the solution of Eq. (43) for underground structures depends only slightly on the value of α. At the same time for α = 0 Eq. (43) can be solved in quadratures, namely,

$$t - \tau = \left(R^2 + R - \frac{6R\ln R}{R-1} + 4\right)\Big/ 12. \tag{47}$$

If one takes into account what was stated above, this result can be employed to determine R(t).

Using the relations (41), (42), and (43) one obtains the following expression for the gradient on the production wall:

$$\left.\frac{\partial F}{\partial r}\right|_{r=1} = -\frac{P}{\dfrac{1}{\alpha} - 1 + \dfrac{R\ln R}{R-1}}. \tag{48}$$

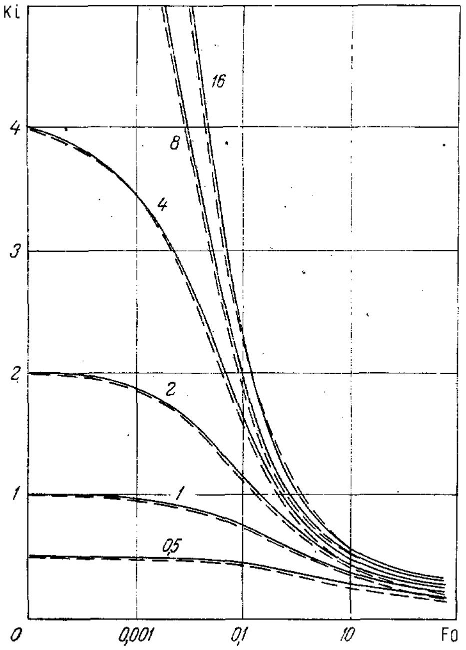


Fig. 1. Error estimation of the method. Dashed lines) exact solutions; solid lines) approximate solutions. Values over curves are the values of $\alpha$.

To estimate the error in the employed method the computation results obtained by using the formula (48) are compared with the exact solution of the problem (29)-(31) in [4]. The comparison is carried out using the Ki criterion of Kirpichev which characterizes the heat flow in the structure wall. Of course, for $P = 1$, one has $(\partial F/\partial r)|_{r=1} = \mathrm{Ki}$. The exact solution given in [4] is in the form of an improper integral with singular points at 0 and at $\infty$. To compute the solution an ALGOL-60 program was prepared for the M-220 electronic computer.

It can be seen from Fig. 1 that there is a good agreement between the exact and the approximate solutions for the values of $\alpha$ under consideration. The relative error never exceeds 3%. It should be mentioned that for higher values of Fo (>100) the exact solutions [4] as computed by us differ considerably (30-50%) from the results of [1].

By inserting the expression (48) into Eq. (28) one obtains an ordinary differential equation of the first order:

$$B\frac{\partial P}{\partial z} = -\frac{P}{\frac{1}{\alpha} - 1 + \frac{R\ln R}{R-1}}. \tag{49}$$

Integrating the equation and using the boundary condition (32), one finds

$$P(z, t, \tau, y) = \delta(t-\tau)\, y \exp\left(-\frac{z}{B\left(1 - \frac{1}{\alpha} + \frac{R\ln R}{R-1}\right)}\right). \tag{50}$$

Hence an approximate solution for $\Pi_i$ is of the form

$$\Pi_i = T_{i0}\left[\delta(t - t_{i-1})\exp\left(-\frac{zp_{i-1}}{B}\right) - \delta(t - t_i)\exp\left(-\frac{zp_i}{B}\right)\right], \tag{51}$$

where

$$p_i = p(R_i) = \frac{1}{\frac{1}{\alpha} - 1 + \frac{R_i \ln R_i}{R_i - 1}},$$

and $R_i$ is a function determined by the relation (47) for $\tau = t_i$. Substituting this expression into (36), one obtains

$$T_1(z,\ t)=\sum_{i=1}^{k} T_{i0}\left[\exp\left(-\frac{zp_{i-1}}{B}\right)-\exp\left(-\frac{zp_i}{B}\right)\right]+T_{(k+1),0}\exp\left(-\frac{zp_k}{B}\right). \quad (52)$$

Let us now consider, in turn, the problem (23)-(27). Solving (24), (26), (27) by integration similarly as it was done above, one obtains

$$\left.\frac{\partial\bar{\theta}_2}{\partial r}\right|_{r=1}=p_0\left(\theta_{\text{M}}-\frac{Q}{B}z-\bar{T}_2\right). \quad (53)$$

Inserting this expression into Eq. (23) one arrives at the following ordinary differential equation:

$$\frac{B}{p_0}\cdot\frac{\partial T_2}{\partial z}=\theta_{\text{M}}-\frac{Q}{B}z-\bar{T}_2. \quad (54)$$

The above equation can be rewritten as follows:

$$\frac{B}{p_0}\cdot\frac{\partial}{\partial z}\left(\bar{T}_2+\frac{Q}{B}z-\theta_{\text{M}}-\frac{Q}{p_0}\right)=\theta_{\text{M}}-\frac{Q}{B}z-\bar{T}_2+\frac{Q}{p_0}.$$

The solution of this equation which satisfies the boundary condition (25) is as follows:

$$\bar{T}_2=\left(\theta_{\text{M}}+\frac{Q}{p_0}\right)\left[1-\exp\left(-\frac{zp_0}{B}\right)\right]-\frac{Q}{B}z. \quad (55)$$

Finally, by inserting the relations (52) and (55) in the expression (11), one obtains after some transformations

$$T(z,\ t)=\theta_{\text{M}}+\frac{Q}{p_0}+\sum_{i=1}^{k+1}A_i\exp\left(-\frac{zp_{i-1}}{B}\right). \quad (56)$$

In the above

$$A_1=T_{10}-\theta_{\text{M}}-\frac{Q}{p_0},\ t_k\leqslant t<t_{k+1},$$
$$A_2=T_{20}-T_{10},$$
$$A_3=T_{30}-T_{20},$$
$$\cdots\cdots$$
$$A_n=T_{n0}-T_{(n-1)0},$$
$$\cdots\cdots$$

In the case of $T_0(t)$ being specified as a continuous function the expression (56) becomes

$$T(z,\ t)=\theta_{\text{M}}+\frac{Q}{p(t,\ 0)}+\left(T_0(0)-\theta_{\text{M}}-\frac{Q}{p(t,\ 0)}\right)\exp\left(-\frac{zp(t,\ 0)}{B}\right)+\int_0^t\frac{\partial T_0}{\partial\tau}\exp\left(-\frac{zp(t,\ \tau)}{B}\right)d\tau, \quad (57)$$

where

$$p(t,\ \tau)=\left(\frac{1}{\alpha}-1+\frac{R(t,\ \tau)\ln R(t,\ \tau)}{R(t,\ \tau)-1}\right)^{-1}.$$

## NOTATION

$\rho_a$, air density; $c_a$, specific heat of air; v, air jet velocity; T, air temperature; $\theta$, rock temperature; $\lambda$, $a$, coefficient of thermal conductivity and thermal diffusivity of rocks; $\bar{\alpha}$, coefficient of heat transfer from the air jet to the inner walls of the mine; $\delta_n$, $\lambda_n$, thickness and thermal conductivity of the n-th insulation layer; $\bar{t}$, time; $\xi$, $\bar{z}$, radial and lengthwise coordinates; $r_0$, radius of mine section; $\bar{\theta}_m$, initial temperature of rock mass; q, power of internal heat sources per meter of mine.

## LITERATURE CITED


1. A. N. Shcherban' and O. A. Kremnev, Scientific Principles of Computation and Regulation of Heat Conditions in Deep Mines [in Russian], Izd. Akad. Nauk UkrSSR (1961).
2. Yu. D. Dyad'kin, Principles of Mining Heat Physics for Mines and Pits in the North [in Russian], Nauka, Moscow (1968).
3. T. Goodman, Trans. Soc. Mech. Eng., 80, 335-342 (1958).
4. H. S. Carslaw and J. C. Jaeger, Heat Conduction in Solids, 2nd ed., Oxford University Press (1959).